ST102 Week 10

Method. (Poisson approximation of Binomial)

Suppose $X \sim Bin(n, \pi)$ & n is large, π small, then distribution of X is well approximated by Poisson $(n\pi)$.

Intuition? On averge & independence.

Why n large? Closer to limiting distribution

Why π small? Var $\begin{cases} \text{Bin}: & n\pi(1-\pi) \\ \text{Poisson}: & n\pi \end{cases}$

Proof ?

Method. (Normal approximation of Binomial)

$Bin(n, \pi)$ close to $N(n\pi, n\pi(1-\pi))$ as $n \nearrow +\infty$, and approximation is better when π away from both 0 and 1. (Usually require $n\pi, n(1-\pi) > 5$)

Intuition? Symmetry & tail behaviours.

Rk. (Continuity correction)

Bin		Normal
$x \in \mathbb{N}$	$\Longleftrightarrow$	$(x-0.5, x+0.5)$

Def. (Multivariate Random Variables)

$$\underline{X} := (\underline{X}_1, \underline{X}_2, \cdots, \underline{X}_n)^T$$

Def. (Joint Probability Function, Discrete)

$$p(x_1, x_2, \cdots, x_n) := \mathbb{P}(\underline{X}_1 = x_1, \underline{X}_2 = x_2, \cdots, \underline{X}_n = x_n)$$

Def. (Marginal Distribution)

$p_I(x_{i \in I})$, $I \subset [n]$

Def. (Joint Probability Density Function, Con't)

Def. (Conditional distribution in discrete bivariate)

$$\begin{aligned} p_{Y|\underline{X}}(y|x) &:= \mathbb{P}(Y = y \mid \underline{X} = x) \\ &= \frac{\mathbb{P}(\underline{X} = x \text{ and } Y = y)}{\mathbb{P}(\underline{X} = x)} \\ &= \frac{p_{\underline{X},Y}(x, y)}{p_{\underline{X}}(x)} \end{aligned}$$

Def. (Conditional Mean & Variance)

$$E_{Y|\underline{X}}(Y|x) = \sum_{y \in \mathcal{D}_y} y \, p_{Y|\underline{X}}(y|x)$$

$$\mathrm{Var}_{Y|\underline{X}}(Y|x)$$

Def. (Con't Conditional Distribution)

$$f_{Y|X}(y|x) := \frac{f_{X,Y}(x,y)}{f_X(x)}$$

Def. (Covariance)

$$Cov(X,Y) = Cov(Y,X) := E\{[X-E(X)][Y-E(Y)]\}$$

$$(\text{linearity}) = E(XY) - E(X)E(Y)$$

Prop. (Properties of Cov.)

i) $Cov(X, X) = Var(X)$

ii) $Cov(aX) = 0$

iii) $Cov(aX+b, cY+d) = ac\, Cov(X,Y)$

Def. (Correlation)

$$Corr(X,Y) = Corr(Y,X) := \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}}$$

(Linear association)