ST102 Week 11

Def. (Independence)

For discrete r.v.'s $\{X_i\}_1^n$, they are independent iff

$$p(x_1, \cdots, x_n) = \prod_{i=1}^{n} p(x_i)$$

for all $x_i \in \mathcal{D}_i$, $i \in [n]$.

For con't r.v.'s $\{X_i\}_1^n$, they are independent iff

$$f(x_1, \cdots, x_n) = \prod_{i=1}^{n} f(x_i)$$

for $\forall$ $x_i \in \mathcal{D}_i$, $i \in [n]$.

Fact. $X \perp\!\!\!\perp Y \Rightarrow Cov(X, Y) = Corr(X, Y) = 0$

Def. (Joint distribution & sum/product of r.v.'s)

Prop. (Linearity of expectation)

$$E(aX + b) = aE(X) + b$$

Fact. (Variance of sum)

$$Var\left(\sum_{i=1}^{n} a_i X_i + b\right) = \sum_{i=1}^{n} a_i^2 Var(X_i) + 2 \sum_{i<j}\sum a_i a_j Cov(X_i, X_j)$$

What if independent ?

Prop. If $\{X_i\}_1^n$ are independent r.v.'s, $\{a_i\}_1^n$ constants, then

$$E\left(\prod_{i=1}^{n} a_i X_i\right) = \prod_{i=1}^{n} a_i E(X_i)$$

Fact. (Sums of independent r.v.'s)

Always assume all r.v.'s are independent:

i) If $X_i \sim Bin(n_i, \pi) \Rightarrow \sum_{i=1}^{m} X_i \sim Bin\left(\sum_{i=1}^{m} n_i, \pi\right)$

ii) If $X_i \sim Poisson(\lambda_i) \Rightarrow \sum_{i=1}^{m} X_i \sim Poisson\left(\sum_{i=1}^{m} \lambda_i\right)$

Prop. (Linear combination of normals)

If $X_i \sim N(\mu_i, \sigma_i^2)$, $i \in [n]$ and $\{a_i\}_1^n$ & b are constants, then

$$\sum_{i=1}^{n} a_i X_i + b \sim N(\mu, \sigma^2)$$

$$\mu = \sum_{i=1}^{n} a_i \mu_i + b \quad \text{and} \quad \sigma^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2 + 2 \sum_{i<j}\sum a_i a_j \, Cov(X_i, X_j)$$