ST102 Week 13

Common sampling distributions

Def. Let $Z_1, Z_2, \cdots Z_k$ be independent $N(0,1)$, if
$$X = \sum_{i=1}^{k} Z_i^2,$$
then $X \sim \chi_k^2$, with degree of freedom k.

RK. $E(X) = k$, $Var(X) = 2k$

Prop. 1) If $X_1, \ldots, X_n$ are independent, with $X_i \sim \chi_{k_i}^2$, then
$$\sum_{i=1}^{m} X_i \sim \chi^2_{\sum_{i=1}^{m} k_i}$$

2) For i.i.d. random sample $\{X_i\}_1^n$ from $N(\mu, \sigma^2)$, and sample variance S^2, then
$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$$

Reason?

Def. (t distribution)

Let $Z \sim N(0,1)$, $X \sim \chi_k^2$, $Z \perp\!\!\!\perp X$, then
$$T = \frac{Z}{\sqrt{X/k}} \sim t_k$$

with degree of freedom k.

Rk. $E(T) = 0$ for $k > 1$;
$Var(T) = \frac{k}{k-2}$ for $k > 2$;
otherwise not exists.

Rk. t_k has heavier tail than $N(0,1)$;
t_k tends to $N(0,1)$ as $k \nearrow +\infty$.

Prop. $\{X_i\}_1^n$ & $\{Y_i\}_1^m$ all i.i.d. from $N(\mu, \sigma^2)$,
then

$$\sqrt{\frac{n+m-2}{\frac{1}{n}+\frac{1}{m}}} \cdot \frac{\bar{X}-\bar{Y}}{\sqrt{(n-1)S_X^2 + (m-1)S_Y^2}} \sim t_{n+m-2}$$

Def. $U \perp\!\!\!\perp V$ with $U \sim \chi_p^2$ and $V \sim \chi_k^2$. Then

$$F = \frac{U/p}{V/k} \sim F_{p,k}$$

i.e., F distribution with degree of freedom (p,k).

Rk. $E(F) = \frac{k}{k-2}$ for $k > 2$,

Prop. 1) If $F \sim F_{p,k} \Rightarrow \frac{1}{F} \sim F_{k,p}$

2) If $T \sim t_k \Rightarrow T^2 \sim F_{1,k}$

3) $\frac{S_X^2}{S_Y^2} \sim F_{n-1,m-1}$

for i.i.d. $\{X_i\}_1^n$ & $\{Y_i\}_1^m$ from $N(\mu, \sigma^2)$