ST102 Week 14

Def. Let $\hat{\theta}$ be an estimator of θ, then its bias is

$$\text{Bias}(\hat{\theta}) := E(\hat{\theta}) - \theta$$

$$\hat{\theta} \text{ is } \begin{cases} \text{positively biased if } \text{Bias}(\hat{\theta}) > 0 \\ \text{unbiased if } \text{Bias}(\hat{\theta}) = 0 \\ \text{negatively biased if } \text{Bias}(\hat{\theta}) < 0 \end{cases}$$

Def. (Mean Squared Error)

$$MSE(\hat{\theta}) := E[(\theta - \hat{\theta})^2]$$

Prop. (Decomposition of MSE)

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + [\text{Bias}(\hat{\theta})]^2$$

Prop. (Linear Transformation)

For independent r.v.'s $Y_1, \cdots, Y_n$ and constants $a_1, \cdots, a_n$, we have

$$E\left(\sum_{i=1}^{n} a_i Y_i\right) = \sum_{i=1}^{n} a_i E(Y_i) \quad ;$$

$$Var\left(\sum_{i=1}^{n} a_i Y_i\right) = \sum_{i=1}^{n} a_i^2 Var(Y_i)$$

Method of Moments Estimation (MME)

Let $\{X_i\}_1^n$ be r.v.'s, i.i.d., from $F(x;\theta)$.
Suppose θ has p coordinates. Denote moment

$$\mu_k = \mu_k(\theta) = E(X^k)$$

and the k-th sample moment

$$M_k := \frac{1}{n}\sum_{i=1}^n X_i^k$$

Then, MME $\hat{\theta}$ of θ is defined as the solution to

$$\{\mu_k(\hat{\theta}) = M_k\}_{k\in[p]}.$$