ST102 Week 15

Settings: given an i.i.d. random sample $\{X_i\}_1^n$ from some population with mean μ and variance σ^2.

Def. The Least Squares Estimator (LSE) of μ is

$$\hat{\mu} := \min_a \sum_{i=1}^n (X_i - a)^2$$

and actually $\hat{\mu} = \bar{X}$.

Fact. $MSE(\hat{\mu}) := E[(\hat{\mu} - \mu)^2] = \frac{\sigma^2}{n}$

Quite often we can only approximate it.

By CLT we know, when n is large,

$$\bar{X} \overset{approx}{\sim} N(\mu, \frac{\sigma^2}{n})$$

Then,

$$P(|\bar{X} - \mu| \leq 1.96 \frac{\sigma}{\sqrt{n}}) \approx 0.95$$

$$\Downarrow$$

$$P(|\bar{X} - \mu| \leq 1.96 \frac{S}{\sqrt{n}}) \approx 0.95$$

Def. The estimated standard error of $\bar{X}$ is

$$E.S.E.(\bar{X}) := \frac{S}{\sqrt{n}} = [\frac{1}{n(n-1)} \sum_{i=1}^n (X_i - \bar{X})^2]^{\frac{1}{2}}$$

Def. Let $\mathcal{L}(\theta) = f(x_1, \cdots, x_n; \theta)$ be the joint prob. (density) function of r.v.'s $\{X_i\}_i^n$. The maximum likelihood estimator of θ is

$$\hat{\theta} = \max_{\theta} \mathcal{L}(\theta)$$

Rk. Log-likelihood : $\ell(\theta) := \ln[\mathcal{L}(\theta)]$

Rk. (Invariance Principle)

If $\hat{\theta}$ is MLE of θ, the $\hat{\phi} = g(\hat{\theta})$ is MLE of $g(\theta)$.

(Not as easy to understand as it looks)