ST102 Week 8

Def. (Discrete uniform distribution)

$$p(x) := \mathbb{P}(X = x) = \begin{cases} \frac{1}{k} & x = 1, 2, \cdots, k \\ 0 & \text{otherwise} \end{cases}$$

$$E(X) = \frac{k+1}{2}, \quad Var(X) = \frac{k^2-1}{12}$$

Def. (Bernoulli) $\quad X \sim \text{Bernoulli}(\pi)$

$$p(x) = \begin{cases} \pi^x (1-\pi)^{1-x} & x = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

$$E(X) = \pi, \quad Var(X) = \pi(1-\pi)$$

$$M_X(t) = (1-\pi) + \pi e^t$$

Def. (Binomial) $\quad X \sim Bin(n, \pi)$

$$p(x) = \begin{cases} \binom{n}{x} \pi^x (1-\pi)^{n-x} & x = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise} \end{cases}$$

$$E(X) = n\pi, \quad Var(X) = n\pi(1-\pi)$$

$$M_X(t) = [(1-\pi) + \pi e^t]^n$$

RK. $Bin(n, \pi)$ is just the sum of n independent trials of $\text{Bernoulli}(\pi)$.

Motivation for Poisson distribution:

1) Prob. of ≥ 2 occurrences at the same time is negligible.
2) Occurrences in any 2 disjoint time intervals are independent.
3) The probability of 1 occurrence in any time interval of length t is λt for some constant $\lambda > 0$.

Def. (Poisson) $\quad X \sim \text{Poisson}(\lambda)$

$$p(x) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} & x = 0, 1, 2, \cdots \\ 0 & \text{otherwise} \end{cases}$$

$(\lambda > 0)$

$E(X) = \lambda$, $Var(X) = \lambda$

$M_X(t) = e^{\lambda(e^t - 1)}$