ST102 Week 9

Def. (Con't uniform distribution) $X \sim \text{Uniform}[a, b]$ $(b > a)$

$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$$

$$E(X) = \frac{a+b}{2} = \text{median}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

Def. (Exponential) $X \sim \text{Exp}(\lambda)$ $(\lambda > 0)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

$$E(X) = \frac{1}{\lambda}, \quad Var(X) = \frac{1}{\lambda^2}$$

$$M_X(t) = \frac{\lambda}{\lambda - t} \quad \text{for } t < \lambda$$

RK. If "#events / unit time" follows Poisson (λ), then the "waiting time between 2 successive events" follows Exp (λ).

Def. (Normal) $\quad X \sim N(\mu, \sigma^2) \quad (\sigma^2 > 0)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[- \frac{(x-\mu)^2}{2\sigma^2} \right] \qquad x \in \mathbb{R}$$

$$E(X) = \mu, \quad Var(X) = \sigma^2$$

$$M_X(t) = \exp\left\{ \mu t + \frac{\sigma^2 t^2}{2} \right\} \qquad t \in \mathbb{R}$$

RK. (Standard Normal)

$Z \sim N(0, 1)$,

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left\{ - \frac{z^2}{2} \right\} \qquad z \in \mathbb{R}$$

$$\Phi(x) := F(Z \le x) = \int_{-\infty}^{x} f(z)\, dz$$

$\uparrow$

Statistical table

Fact. 1) $Y = aX + b \sim N(a\mu + b, a^2\sigma^2)$ for $X \sim N(\mu, \sigma^2)$

2) For $X \sim N(\mu, \sigma^2)$, always have

$$Z := \frac{X - \mu}{\sigma} \sim N(0, 1)$$